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EFFECT OF DENSITY MODEL TIME-DELAY ERRORS ON ORBIT PREDICTION

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This study examines the effects of density model time-delay errors by focusing on the CHAMP spacecraft data over the 2003-2007 time period. The data is first analyzed to determine the magnitudes of time delays encountered when using a density model. The analysis is initiated by examining the effects of the expected delays of one to three hours on a theoretical density profile. Delays are then introduced into the model and actual density profile for multiple different cases, and the effects on the orbit prediction results are examined. It is found that the effects are potentially significant for the applications of interest in this study.

INTRODUCTION

Significant improvements in atmospheric density models over the last several decades have translated directly into corresponding advances in the quality of orbit predictions. New requirements for improvements in orbit predictions have since made clear the need for further refinements in density models, which is the focus of the current Neutral Atmosphere Density Interdisciplinary Research (NADIR) effort at the University of Colorado, Boulder. One question that naturally arises as this problem is approached is which improvements in density models result in the greatest improvements in orbit predictions. Answering this question can guide the efforts of researchers as they further refine these atmospheric models. In Anderson, Born, and Forbes the effects of modeling spatial variabilities, and more specifically various horizontal wavelengths, in the atmosphere model on orbit prediction results were examined.[1] The effects of spatial errors in the models were quantified using theoretical models as well as actual satellite data. Early comparisons of the effects of temporal errors in the density models with spatial errors have indicated that time-delay errors are potentially more significant. Consequently, quantifying the effects of time-delay errors has been taken as the focus of this study.

Current atmospheric density models have generally been sufficient to meet orbit prediction requirements, but they have proven inadequate for some situations. One such situation occurred during the March 14, 1989 magnetic storms where difficulties were encountered tracking multiple satellites. The difficulties in this case were primarily a result of the inability of the atmospheric

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models to predict the density variations during the magnetic storm. A summary of some of aspects of the accuracy of current density models may be found by referring to Marcos[2] and Marcos et al.[3] A dramatic illustration of the time-delay effect being studied here may be seen by referring to Forbes.[4] In 1972, Forbes examined the effect of time-delay errors in the density model on the prediction of the location of the DB-7 satellite, and found that a premature density increase predicted by the Jacchia model[5] actually resulted in as large of an error as simply predicting no variation in the density at all. While the quarter day time difference between the predicted and actual peak for this case is no longer observed, 1-4 h delays with potentially very significant effects on orbit prediction are still possible. Examining delays of this magnitude is the focus of this analysis.

Before examining the effects of time delays on orbit prediction, it is important to determine what orbit error magnitudes are considered significant. The U.S. Air Force and others have focused on improving their density models for orbit prediction for quite some time,[6,7,8,9,10,11,12,13] and the primary motivation for improvements in the density model in this study is tied to the U.S. Air Force orbit prediction requirements. A summary of these requirements for several altitudes is reproduced from Anderson et al. in Table 1.[14] Comparing orbit prediction results to these requirements will be

Table 1. 24-hour orbit prediction errors of significance to the U.S. Air Force for several altitudes.

Altitude (km)	Error (m)
200	> 250
400	> 100
800	> 50

the primary focus of this paper, but it is important to remember that other applications may possess more stringent requirements.

This study begins by looking at time delays based on the fact that atmospheric models use various measured parameters in an attempt to model the real-world atmospheric densities as accurately as possible. This reliance on real-world parameters makes it possible for these density predictions to lag up to several hours behind the actual onset of a storm, meaning that a density predicted at a particular time may then potentially deviate greatly from the actual atmospheric density at that time. This problem is analyzed here by first examining the statistical differences between the model and actual data to determine the model's effectiveness at predicting densities during both storm times as well as quiet times. For this, the paper uses densities computed from observations of the Challenging Minisatellite Payload (CHAMP) satellite as truth. The NRLMSISE-00 empirical atmosphere model with atmospheric densities computed from time, location, altitude, ap daily index values, and F10.7 daily solar flux values is used as the model. The maximum difference in amplitude between predicted and truth densities is examined to quantify the model's accuracy. The amount of time required for the model to predict the observed densities is also analyzed. It is known that the model typically lags behind during storm times, but it is also necessary to examine the model's behavior during quiet times. Initially, though, the problem is approached by examining the effects of delays in specified density profiles. The difference in a satellite's orbit is then quantified by performing numerical simulations using the model and truth densities. Prediction errors comparing orbits integrated with actual density profiles and those integrated using a model with perfect inputs are compared first. Then comparisons of cases using nominal model densities are performed and model densities with a delay are introduced into the inputs. In this case, the ap values used as inputs are delayed by up to several hours, which forces the model to predict densities later than the truth densities. Finally, comparisons are made using a smoothed version of the density data and a delayed version of this data. In each of the cases, simulations are performed for both quiet and active times, and an analysis of how the results change with altitude are included. The resulting errors for each step are quantified and placed within the context of potential orbit prediction requirements.

MODELS

The focus for this study is quantifying the representative effects of errors in the density model for a spacecraft in various types of low Earth orbits. Therefore, it was decided to use a simple two-body model for the spacecraft integration with a detailed atmospheric model for computing the density and drag on the spacecraft. This choice allows the study to focus on the desired effects of the time delays in the density model and eliminates the need to account for the effects of detailed gravity models. The specific models used for the computations are discussed next.

Two-Body Model with Drag

Acceleration due to drag (or force per unit mass) is

$$\bar{a}_{drag} = -\frac{1}{2}\left(\frac{C_d A}{m}\right)\rho V_a \bar{V}_a \tag{1}$$

where $m/(C_d A)$ is referred to as the ballistic coefficient. The velocity relative to the atmosphere may be computed as

$$\bar{V}_a = \begin{bmatrix} \dot{x} + \dot{\theta} y \\ \dot{y} - \dot{\theta} x \\ \dot{z} \end{bmatrix}. \tag{2}$$

Anderson, Born, and Forbes[1] examined prograde, retrograde, and polar orbit cases and found that the results of interest to this work varied little with the orbit type. The results shown here are therefore all generated using polar orbits unless otherwise specified.

Density Model

The primary density model used in this research is the U.S. Naval Research Laboratory Mass Spectrometer and Incoherent Scatter Radar (NRLMSISE-00) atmospheric density model.[15] This model is the same one used by Sutton to generate the model densities provided with the CHAMP data.[16] Inputs to the model include basic information such as time, location, altitude, ap daily index values, and F10.7 daily solar flux values. For this study, the desired output was the atmospheric density at the specified location. The accuracy varies, of course, depending on whether the input parameters are predictions or computed after the fact.

CHALLENGING MINISATELLITE PAYLOAD

Spacecraft

This analysis focuses on the CHAMP spacecraft. The CHAMP spacecraft was launched into a low Earth orbit on July 15, 2000. Its initial altitude was 454 km, and it had an inclination of approximately 87 degrees. Its total mass at launch was 522 kg, which dropped to approximately 505-507 kg in 2003. A more detailed description of the CHAMP spacecraft and its mission may be found in Reigber et al.[17] and Kuang et al.[18]

Data

This research uses thermosphere density data obtained from the sensitive accelerometer aboard the CHAMP spacecraft. Sutton has processed this data and made it available at a website at the University of Colorado, Boulder.* For additional details about the application of this data and its processing refer to Sutton et al.[19,20,21] or Sutton's dissertation.[16] The data for the CHAMP spacecraft is available from 2001 through 2008, and the analysis here focuses on the years from 2003 through 2008. This allows the results to cover time periods including the large geomagnetic storm of 2003 through the geomagnetically quieter years into 2008. Note that the density values have been reprocessed and updated since the results were computed for Anderson et al. in 2009. The version 2.2 CHAMP densities are used for this analysis.

Storms in the Data

Several specific storms were chosen for more in-depth analysis in this paper in order to determine the sorts of time delays that may exist in the model. These storms were selected by using the geomagnetic indices to pinpoint some of the times of interest. The K-index is a measurement used to quantify the disturbances in the horizontal component of Earth's magnetic field. K-index values are computed on a daily three-hour basis at several locations around the world. The planetary Kp index is computed through a weighted average of the K-index values from the various observatories. The ap index is a linear index related to the Kp index as shown by the scale in Table 2. The ap index

Table 2. Relationship between Kp and ap.

Kp	ap	Kp	ap
$0_\circ$	0	$5.$	39
0_+	2	$5_\circ$	48
$1.$	3	5_+	56
$1_\circ$	4	$6.$	67
1_+	5	$6_\circ$	80
$2.$	6	6_+	94
$2_\circ$	7	$7.$	111
2_+	9	$7_\circ$	132
$3.$	12	7_+	154
$3_\circ$	15	$8.$	179
3_+	18	$8_\circ$	207
$4.$	22	8_+	236
$4_\circ$	27	$9.$	300
4_+	32	$9_\circ$	400

is useful when sums and averages of daily activity are desired. See Prölss[22] for more information on Kp and ap as well as for a source of the values in Table 2.

*Data available online at http://sisko.colorado.edu/sutton/

Geomagnetic storms and their strengths can be classified by Kp value, with NOAA having five different classifications for a storm.[23] The storm classification begins at a Kp value of 5 and increases in severity as the Kp value rises, with 9 being the most intense storm. During active years, Kp values of 5 or greater can be quite common. In 2003, there were 103 days in which a Kp value of at least 5 was observed. More severe storms, as classified by a higher Kp value, are less common. A Kp value of 9 is observed only twice in the years studied, occurring during the same storm in 2003, which was the most active year. The number of storm days for the years 2004 through 2008 is significantly less, but a Kp value of 8 is still observed on nine days during that time period. Table 3 gives the precise number of days where each Kp value occurred throughout the studied years.

Severe magnetic storms and their effects on neutral density are of interest to this study. Thus, four storms were selected for detailed study based on the observed Kp values. The first selected storm occurred on October 29, 2003, and Kp values as high as 9 were observed during this storm. The next selected storm occurred on November 23, 2003, and the Kp index during this time period reached values up to 8. The final two storms occurred in the less active year of 2004, though the geomagnetic activity in 2004 was greater than what is seen in the following years, meaning significant storms are still observed. The two storms in 2004 occurred on July 27 and November 15. Once again, the Kp index reached values of up to 8 during this period.

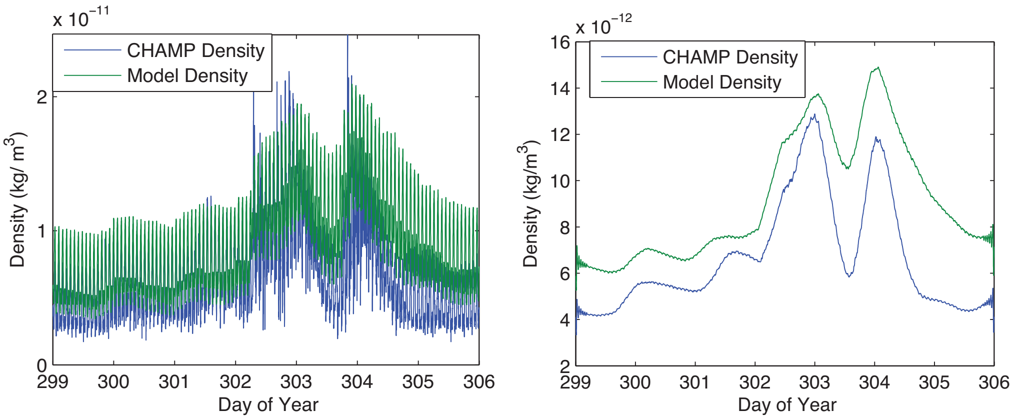
Table 3. Number of days that Kp exceeded the specified value each year.

	Kp Cutoff Values				
Year	5	6	7	8	9
2003	103	28	10	5	2
2004	27	14	9	5	0
2005	46	19	14	3	0
2006	26	8	3	1	0
2007	18	0	0	0	0
2008	10	2	0	0	0

Time Delays in the Data

Measuring the time delays in the model density for several storms provides an understanding of the types of time delays that may be expected and provides values to focus on for the remainder of this study. Sutton also provides some analysis of the data that gives an indication of the types of errors that might be expected for storms or other density increases. The storms discussed next were chosen to obtain a representative sample of the general response of the NRLMSISE-00 model to changes in geomagnetic activity. For these cases, the approximate time that it took the model to display a peak density as compared to when the CHAMP data showed a peak density was examined. The model density was computed using the position of the CHAMP satellite as given by Sutton. In order to determine when the peaks in the density occurred, both sets of data were smoothed using 701 points. By doing this, it is possible to determine the time delay between the model and the observed density by examining the plots shown in Figures 1 and 2.

The actual densities and the smoothed densities in Figure 1 show the differences that can be expected between the measured and modeled densities. The model density was larger over this



(a) Actual Density (b) Smoothed Density

Figure 1. October 29, 2003 plot showing actual density and smoothed density.

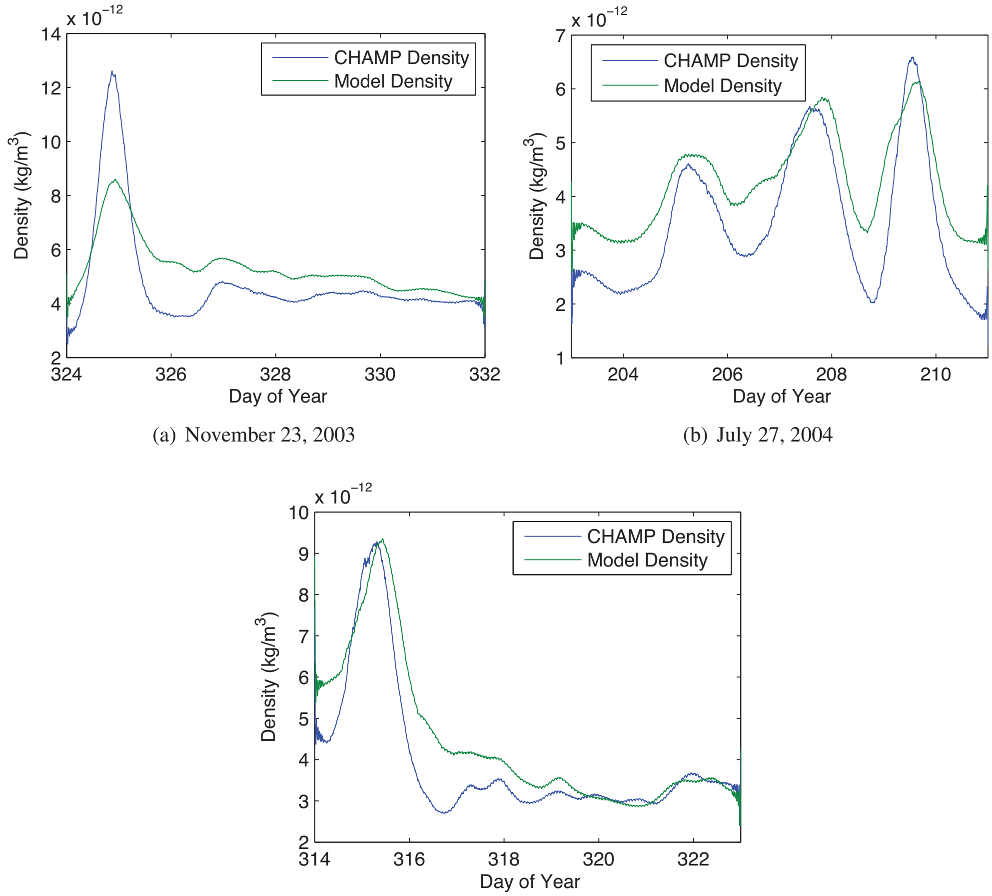
time period, but the primary parameter of interest here is the time delay. It is difficult to pick out a time delay using the actual densities, but the smoothed densities show a time delay, especially near the first major peak. Even longer delays can be observed in the smoothed densities in Figure 2. Some of the larger delays are seen here for the July 27, 2004 case. By examining these delays and others through the time period of interest, it was determined that in general the model density can lag behind the actual density by anywhere from 1.5 to 3 hours. These time delays were used as a reference when performing the simulations discussed later in the paper.

EFFECT OF TIME-DELAYS ON ORBIT PREDICTION

Short-Term Time Delays using an Analytical Density Model

Some insights into the types of orbit prediction errors that might be expected from time delays in the atmosphere model may be gleaned by examining the effects of a time delay in an analytical atmosphere model. By using a simple analytical model, the effects of the time delay may be isolated, and the characteristics of the density perturbation, such as amplitude, may be specified. Anderson et al. showed that the primary consideration in modeling a density function was the imparted impulse and that functions of different shapes with the same imparted impulse will result in similar orbit changes. As a result, this study uses half of a sine wave over half of an orbit to model an increase in the density. To determine the effects of a time delay, two spacecraft were integrated forward from the same initial conditions. The first spacecraft encountered the specified perturbation, and the trajectory was then integrated for a total of 24 hours. The second spacecraft encountered the same perturbation, but at a later specified time. See Figure 3 for a schematic illustrating the trajectories encountering the density perturbations at different times.

Different cases were computed while varying the time delay up to four hours. The amplitude of the variation was also changed up to 200 percent of the nominal density at the specified altitude, which for these simple cases was computed using the 1976 standard atmosphere.[24] The results are plotted in Figure 4 for altitudes of 400 km and 700 km. The effects at an altitude of 400 km sometimes approach over 20 m, which could be noticeable for some applications, while the effects



(a) November 23, 2003

(b) July 27, 2004

(c) November 15, 2004

Figure 2. Plot showing smoothed densities from CHAMP versus model densities for three sample time periods.

at 700 km are relatively minor. Keep in mind that these effects would scale with area or the ballistic coefficient, so even these small errors may become of significance relative to the U.S. Air Force requirements if the area of the spacecraft were to become much larger.

The effect of varying the altitude on these results may be further examined by computing the position differences for a specific case over several altitudes. This process was done for a specific case with a time delay of three hours and a density perturbation amplitude of 200 percent in Figure 5. The position differences decrease dramatically at 500 km to approximately 5 m and further down to approximately 1 m at 600 km. Plotting the results compared to the density reveals that they closely follow the value of the density at the specified altitude as would be expected.

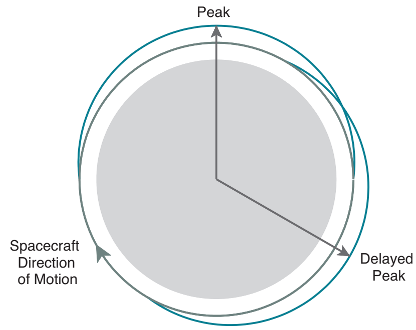
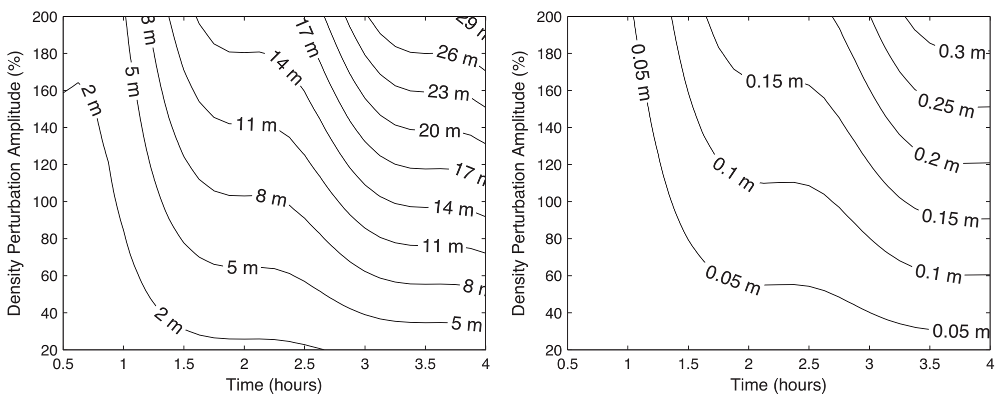


Figure 3. Schematic illustrating the two cases with one spacecraft trajectory encountering a peak in the density at the actual time and another spacecraft encountering the peak at some delayed time.



(a) 400 km

(b) 700 km

Figure 4. Final positions differences after a 24 hour integration for different combinations of time delays and density perturbation amplitudes. In each case, the perturbation is modeled so that it covered half of a revolution.

Orbit prediction errors with perfect model inputs

It is necessary to examine the total error arising from the use of density models to represent the actual atmosphere in order to determine the importance of the time-delay errors relative to this total error. The orbit errors arising from the use of the model densities in place of CHAMP densities were examined briefly for 2003 and 2007 in Anderson et al. This problem is approached here again for the years from 2003 through 2008 using the updated CHAMP densities. The inputs to the density model used here are the best available assuming no timing delays. The introduction of time delays to the process will be carried out in the following section.

An idea about the accuracy of the model densities may be obtained by comparing the CHAMP derived densities to the NRLMSISE-00 model densities provided by Sutton in the same data. The differences between the two density values are plotted in Figure 6. It can be seen from the plot that the largest density excursions occur near the 2003 geomagnetic storm. The differences generally fluctuate around zero, although the density seems to deviate for more recent years.

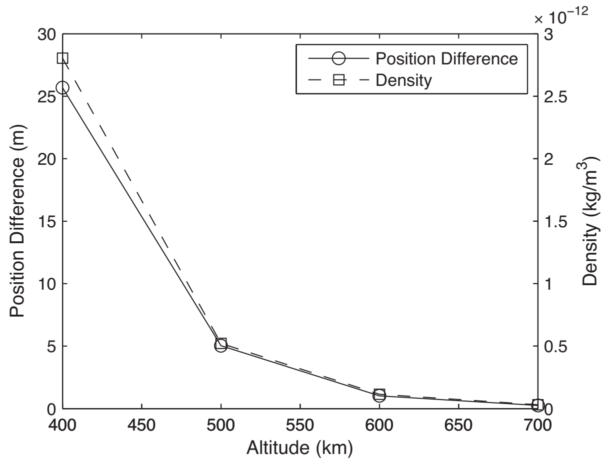


Figure 5. Position differences after a 24 hour integration at various altitudes for a time delay of three hours and a density perturbation amplitude of 200 percent.

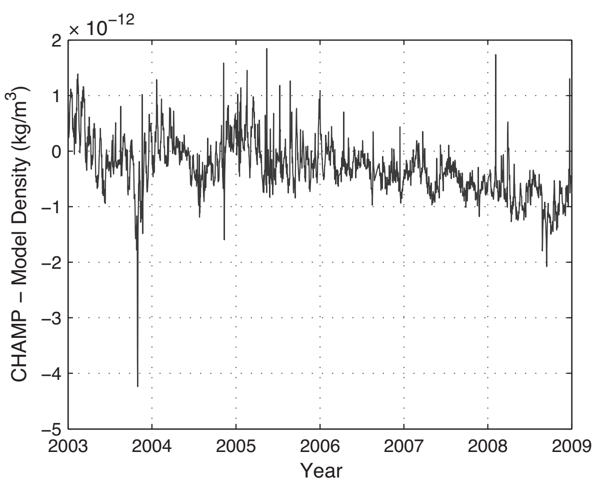
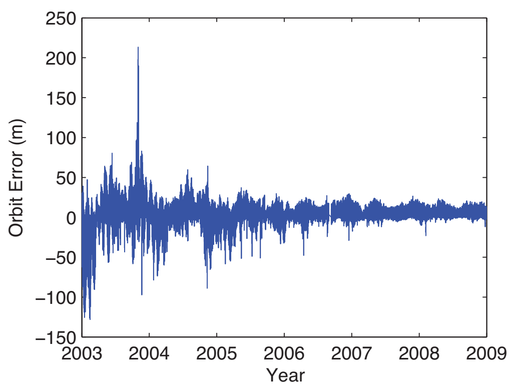
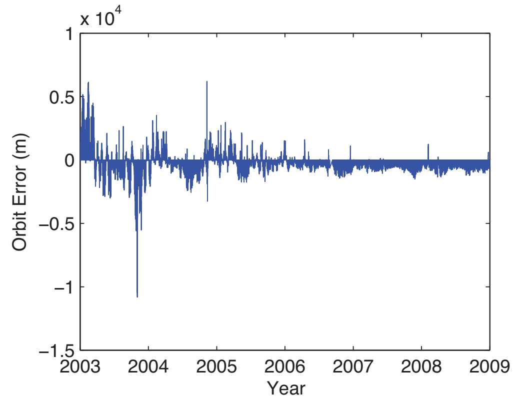


Figure 6. Difference between mean densities computed for each day at the CHAMP satellite's altitude.

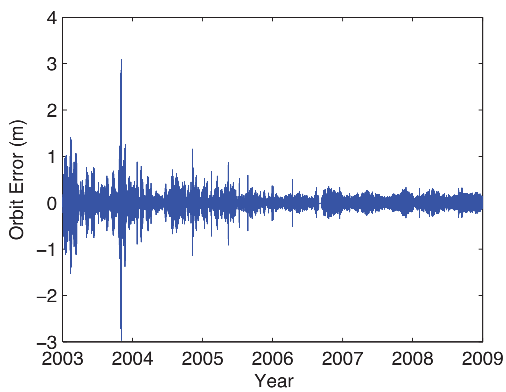
It is straightforward to numerically compute the effect of the differences in the densities on the spacecraft's trajectory. Specifically, a simulation was performed to compare the effect of using perfect model data and the CHAMP data in orbit propagation. For this simulation, a theoretical spacecraft with a mass of 500 kg and a polar circular orbit with a starting altitude of 400 km was assumed. The equations of motion for the spacecraft assume only drag, and a coefficient of drag of 2 was selected. The spacecraft state was then integrated over 24 hour intervals for the years 2003 through 2008. The simulation was performed twice. The first simulation was performed using the densities observed by CHAMP, which were mapped to 400 km using the NRLMSISE-00 model. The second simulation was performed using the densities computed by the NRLMSISE-00 model computed at 400 km. When using the NRLMSISE-00 model, perfect inputs are assumed, meaning



(a) Radial



(b) In-Track



(c) Cross-Track

Figure 7. Theoretical spacecraft orbit differences computed using CHAMP density values and NRLMSISE-00 model density values. The orbit errors are computed after a 24 hour integration.

that the values used for the ap planetary index and the F10.7 daily solar flux are the exact values seen that day and not estimated values.

The differences in the spacecraft orbit are computed in the radial, in-track, and cross-track directions and are plotted in Figure 7. It can be seen that the simulation agrees with what was previously examined by Anderson et al. for the years 2003 and 2007, where 2003 is a geomagnetically active year and thus has larger differences in the orbits, while 2007, which was a much quieter year has significantly smaller orbit differences. Of the years studied, 2003 was the most active, and the subsequent years became increasingly quieter in terms of solar activity. As such, 2003 shows the greatest differences in the satellite orbit predicted using the CHAMP density data compared to the orbit that used the model densities.

Again, in accordance with what was seen by Anderson et al., orbit differences are largest in the in-track direction. While the radial direction shows significant differences up to 140 m, the differences in the in-track direction can be measured at the kilometer level. As stated previously, the largest differences occur in the years that have high solar activity, and the in-track difference at one point reaches a value of almost 8000 m. This significant difference occurs during the large October 2003 geomagnetic storm, where a Kp value of 9 was observed. Orbit differences of greater than 4000 m are never seen other than during the October 2003 storm. During the years 2004 through 2008, the orbits agree within 2000 m except on one occasion in 2004. This occurrence again corresponds to a large geomagnetic storm, though it is not as large as the previously mentioned storm in 2003. The years 2006 through 2008 have very little solar activity and thus show the smallest orbit differences.

Orbit prediction errors with delays in the model

While using perfect model inputs is obviously the preferred method for post processing, as orbit prediction results are being generated, some delay is often introduced in the model inputs. In the previous section, the best known parameters were used after the fact. Now time delays will be introduced into the density input parameters for the model, and the effects on the orbit prediction results will be analyzed. The focus here will be on the year 2003 in order to capture the larger effects for a geomagnetically active year. Results for 2004 will also be given for comparison of the results with a less active year.

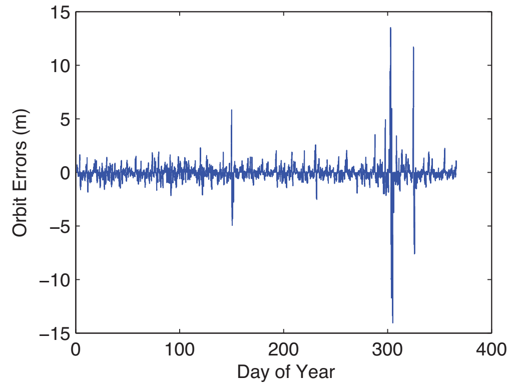
Specifically, the effects of time delays on orbit propagation were examined using three additional simulations in which imperfect inputs to the NRLMSISE-00 model were used. To simulate the delays in the modeling of the density, the ap values used in the model are delayed by a specified number of hours. The simulations performed assumed delays of one, two, and three hours. These values correspond to the range observed previously by examining the time it took for the model density to reach its peak during storms. The simulations were performed with and without the delayed densities, and the states after a 24 hour integration were compared.

The results are shown in Figure 8 for the one and three hour cases over 2003. Note that the cross-track direction differences are not displayed because the errors are on the meter level, which is not significant compared to the errors seen in the other two directions. The two hour results are not plotted here, but they possess trends similar to the three hour case. In general, the delays were found to increase linearly for each hour increase in delay.

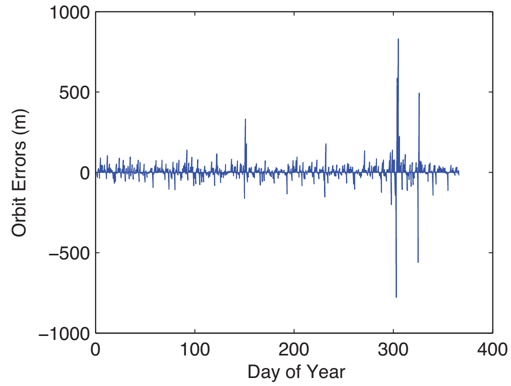
The statistical results from the data are given in Tables 4 and 5. Table 4 contains the mean values for the errors in all directions for the three different delays. While the errors in all directions remain relatively small throughout the year, maximum values can reach the kilometer level, which is very significant. Also, the mean values can reach significant values in the tens of meters. Table 5 contains the standard deviations for the errors, and as expected, the largest occur in the in-track direction. The three-hour delay case has a deviation over 100 m, which is significant to the Air Force at the satellite altitude of 400 km.

Table 4. Mean of the maximum orbit differences along the final orbit after a 24-hour integration for 2003. The difference is between the nominal model and delayed model cases.

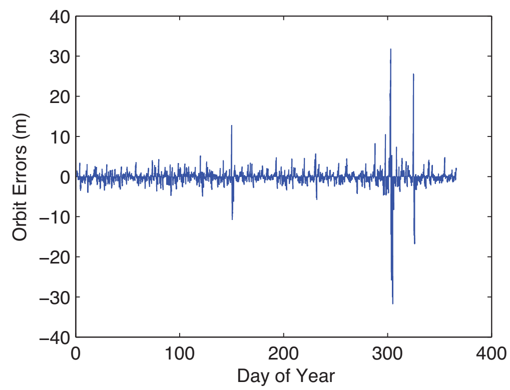
Time Delays (h)	Radial (m)	In-Track (m)	Cross-Track (m)
One	0.382	18.08	0.0043
Two	0.705	32.92	0.007
Three	0.998	46.09	0.0104



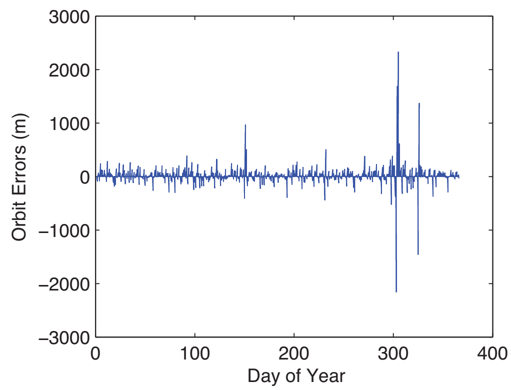
(a) Radial (1 hour)



(b) In-Track (1 hour)



(c) Radial (3 hour)



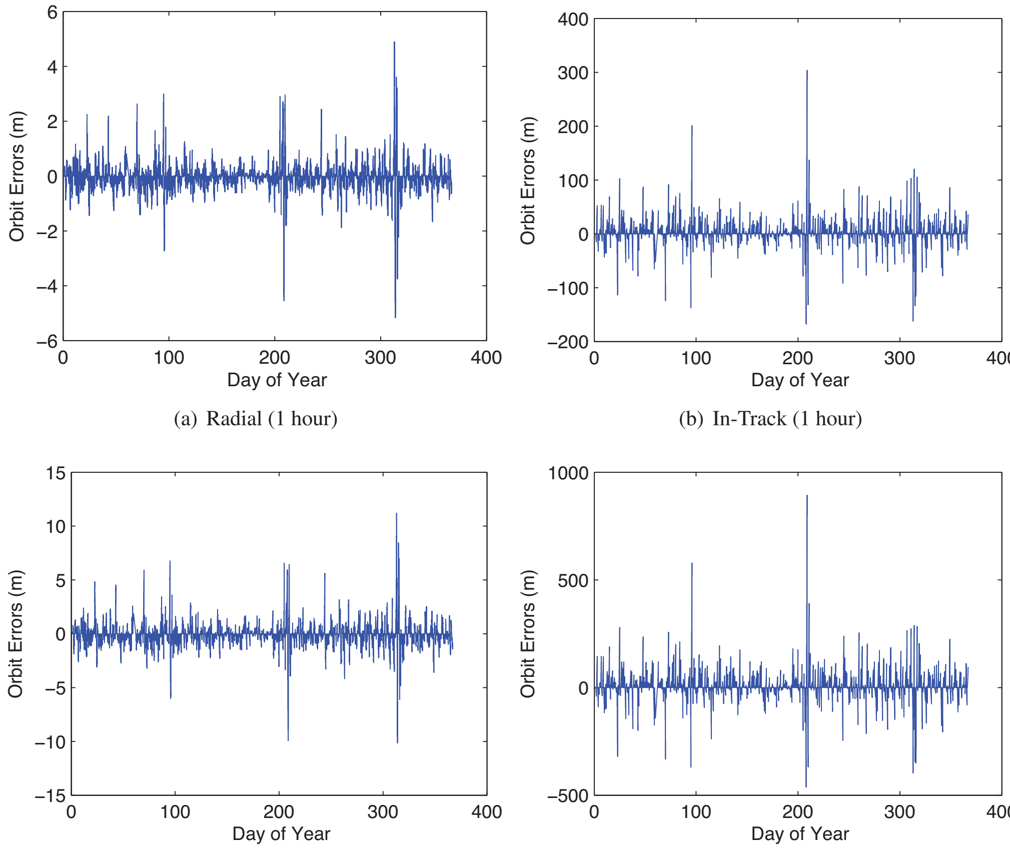
(d) In-Track (3 hour)

Figure 8. Orbit prediction errors for one and three hour model delays over 2003.

Table 5. Standard deviations of the orbit errors after a 24-hour integration for the delayed model given in Table 4

Time Delays (h)	Radial (m)	In-Track (m)	Cross-Track (m)
One	0.8645	46.5112	0.0106
Two	1.5892	86.0126	0.0197
Three	2.2491	121.4777	0.0263

A simulation comparing time delays was also performed for 2004 in order to compare the storms to those in 2003. As stated previously, this year provides an intermediate level of geomagnetic activity for study. Figure 9 displays the radial and in-track errors for the one and three hour delays. Once again, the most significant errors occur in these directions, while the cross-track errors are less than a meter.



(a) Radial (1 hour) (b) In-Track (1 hour)

(c) Radial (3 hour) (d) In-Track (3 hour)

Figure 9. Orbit prediction errors for one and three hour model delays over 2004.

The largest errors in 2004 occur during the storm in July, where Kp values up to 8 were observed. This storm led to the largest errors in the in-track direction, which is approximately 900 m for the three hour delay. The smaller delays also have their largest delays at this time but with smaller magnitudes. The other large difference in the in-track errors occurred in April, where the error reached nearly 600 m for the three hour delay. This is not one of the storms that was examined earlier. In fact, the Kp index only showed values of up to 6 during this time period. The other storm examined, which occurred in November and did reach Kp values of 8, does show a large error in the in-track direction, but it is smaller than what was seen in April. This is significant in that it shows that even less severe storms can cause significant errors in orbit determination. The mean values and standard deviations for each of the cases are displayed in Tables 6 and 7. As was seen with the 2003 case, the orbit errors are typically small and not of significant levels throughout the year, although there are exceptions where the error can reach several hundreds of meters. The means for 2004 are less than 30 m for each of the delays, meaning that they are less than the errors which are significant to the Air Force. The standard deviations are also less than the significant values for the Air Force, although they can reach up to 50 m in the in-track direction for the three hour delay case. While this may indicate that the errors will not typically be significant for a year with geomagnetic activity similar to what occurred in 2004, the extreme values are significant enough to merit attention. Also, one must keep in mind that these values scale with altitude and spacecraft size, so these values may be significant for a different spacecraft.

Table 6. Mean of the maximum orbit differences along the final orbit after a 24-hour integration for 2004. The difference is between the nominal model and delayed model cases.

Time Delays (h)	Radial (m)	In-Track (m)	Cross-Track (m)
One	0.242	11.17	0.0026
Two	0.462	21.09	0.0048
Three	0.659	29.76	0.0064

Table 7. Standard deviations of the orbit errors after a 24-hour integration for the delayed model given in Table 6

Time Delays (h)	Radial (m)	In-Track (m)	Cross-Track (m)
One	0.349	18.4787	0.0039
Two	0.644	35.3974	0.0077
Three	0.9151	50.6429	0.0105

Orbit prediction errors with delays in smoothed CHAMP density data

The previous sections give an idea of the actual sorts of errors that can be encountered using models that include time delays. The effect of time delays for a specific type of analytical density perturbation of a particular duration has also been examined. It is now desirable to isolate the effect of a time delay but to do this using real-world density fluctuations rather than analytical models.

This type of analysis is implemented here by starting with the actual CHAMP density values mapped to an altitude of 400 km. This data contains variations due to the location of the CHAMP spacecraft as it travels through its orbit in addition to the larger variations in density over time arising from geomagnetic disturbances and storms. In this case, for a spacecraft in a simple circular orbit using a two-body model, it is desirable to isolate the variations in the density arising from the geomagnetic storms. This goal is achieved here by simply smoothing the data using 701 points. This process results in a relatively smooth density profile that eliminates short term density variations but still retains the density variations arising as a result of geomagnetic storms.

A similar process to that used for the analysis of time delays in the analytical density perturbations is used to analyze time delays here. In this case, one spacecraft is integrated from circular orbit initial conditions through the given smoothed density profile for the chosen day. This initial density profile is defined to be the truth density profile for the purposes of this study. An identical spacecraft is integrated through the smoothed density profile delayed by a selected time period. The resulting differences after 24 hours are then recorded.

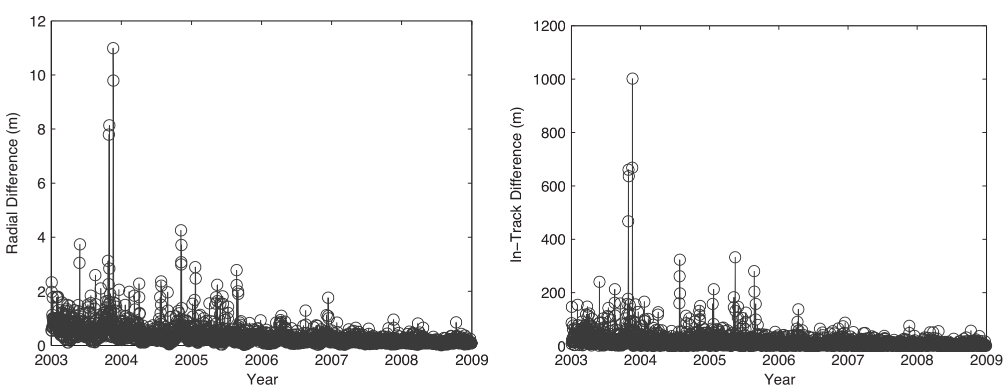
The process described above was used to compute the position differences resulting from time-delay errors in density for cases with one, two, and three hour time delays. The results were computed for every day from 2003 through 2009, and the orbit differences in the radial and in-track directions are plotted for the selected time delays in Figures 10 through 12. The differences were computed for the end of the 24 hour integration. Specifically, the maximum difference was computed over the last orbital period and compared to the minimum difference, since it is expected that this difference would vary over the orbit. In general the difference between the maximum and minimum orbital differences varied by approximately 10 to 20 percent, so only the maximum orbital differences are plotted in the figure. Note that the cross-track differences were generally significantly less than a meter, so they are not plotted here.

Table 8. Mean values of the maximum difference for the final orbit after a 24-hour integration.

Time Delays (h)	Radial (m)	In-Track (m)	Cross-Track (m)
One	0.395	20.58	0.0041
Two	0.63	40.92	0.0081
Three	0.79	61.0	0.012

Table 9. Standard deviation values of the maximum difference for the final orbit after a 24-hour integration

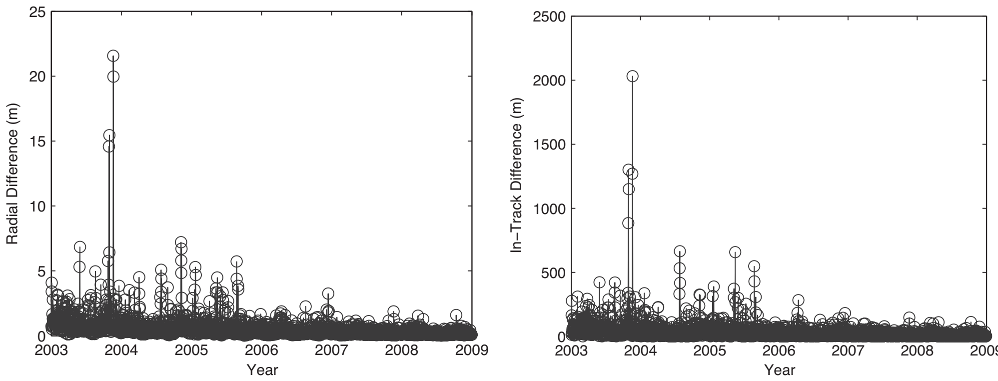
Time Delays (h)	Radial (m)	In-Track (m)	Cross-Track (m)
One	0.553	43.72	0.0081
Two	1.053	86.071	0.0162
Three	1.530	125.94	0.0241



(a) Radial (b) In-Track

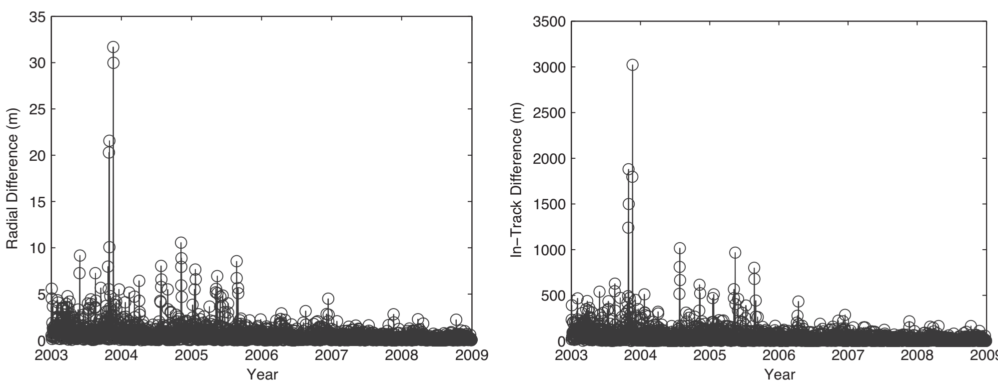
Figure 10. Orbit differences for each day comparing an orbit with a one-hour time delay in the density profile over a 24-hour integration.

Examining the differences for the one-hour case in Figure 10 reveals, that as might be expected, the largest differences occur for days where the geomagnetic activity is very high. The radial differences are two orders of magnitude less than the dominant in-track differences. The maximum in-track differences for the storm days are often higher than the 100 m error limit at a 400 km altitude that would start to be of concern. The average in-track difference of 20.58 m given in Table 8 is less than that limit, but it is worth examining longer time delays.



(a) Radial (b) In-Track

Figure 11. Orbit differences for each day comparing an orbit with a two-hour time delay in the density profile over a 24-hour integration.



(a) Radial (b) In-Track

Figure 12. Orbit differences for each day comparing an orbit with a three-hour time delay in the density profile over a 24-hour integration.

The radial and in-track orbit differences for the two-hour time delay are shown in Figure 11 for the 2003 to 2008 time span. Examining the figures reveals that the differences appear to be approximately twice those of the one-hour time span. The maximum in-track difference is approximately 2000 m compared to 1000 m for the one-hour time span, and the radial differences show a similar effect. The mean for the two-hour case in Table 8 is approximately twice that of the one-hour mean for the radial, in-track, and cross-track cases. The mean radial values rise to 40.92 m, and the standard deviation of approximately 86 m in Table 9 for this case is becoming significant.

The results shown for the three-hour time delay case in Figure 12 continue the trend with values approximately three times those of the one-hour case. The mean value of 61.0 m and the standard deviation value of 125.94 m in the in-track direction cross into the levels of concern to the Air Force. In general, the standard deviations given for each case in Table 9 follow a similar trend with corresponding increases with each increment in the delay time. It can be seen that the in-track standard deviations are approximately twice the mean values in each case. It is apparent from these results that a significant number of cases will most likely fall over the acceptable range given in Table 1 for the 400 km altitude, especially when more geomagnetically active years are considered. This result is especially true given that from previous studies, the orbit differences are expected to scale with the area.

CONCLUSION

Several different approaches were used to measure the effect of time delays in the density model on orbit predictions. It was determined that neutral density predictions can lag behind the true density by several hours during times of severe geomagnetic activity and geomagnetic storms, and a range of one to three hours was found to be typical. An analytical analysis showed that a delay in a single density perturbation could cause errors on the order of 10 to 20 meters for a typical satellite at an altitude of 400 km.

Larger effects were observed when comparing orbit prediction results from a simulation using model densities to one using model densities with delayed inputs. In the year 2003, which was the most geomagnetically active of the years studied, the maximum errors were seen to be over 2 kilometers. These errors are significant enough that they can cause major problems in satellite operations during storms. For 2003, the mean maximum in-track errors varied from approximately 18 m to 46 m linearly with the time-delay. With standard deviations varying from 46 m to 121 m, the potential orbit errors are in the range considered important to the Air Force.

Similar effects were observed when the orbit prediction results from a smoothed version of the actual data were compared to those obtained using a delayed version of the same data. For this case, the results were examined from 2003 through 2008 with the years following 2003 having reduced geomagnetic activity. As expected, the orbit errors decreased as the years progressed. Despite this, the maximum errors were still significant, as they could reach hundreds of meters even during the less active years. The mean maximum errors in this case varied in the in-track direction from approximately 20 m to 61 m, with the standard deviations varying from 44 m to 126 m. These values are again of significance to the Air Force, and they lie in the same range as the values obtained using the delayed model results. This result obtained using different techniques indicates that these numbers are indicative of what might be expected from time-delay errors in the density model. Time delays in the density model are a significant factor in orbit prediction, and they were shown to contribute to a large portion of the total error in the orbit prediction. Any efforts to reduce the time-delay errors in the density models would be expected to result in corresponding improvements in the orbit predictions.

FUTURE WORK

There are several potential avenues for future work. One possible area of interest to examine would be using a more robust method for predicting the satellite's orbit. The gravity model used in this study was simplified to focus on the effects of time delays, but a more accurate model would allow further comparisons with actual satellite data. Previous work has shown that the effects found in this analysis should scale with area, but it would be interesting to examine satellites with different ballistic coefficients directly in addition to looking at more altitudes. Finally, different density models could be tested. One candidate is the Jacchia-Bowman 2008 model which uses different inputs than the NRLMSISE-00 model.

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